2-1 Relations and Functions

Main Ideas

- Analyze and graph relations.
- Find functional values.

New Vocabulary

ordered pair
Cartesian coordinate plane
quadrant
relation
domain
range
function
mapping
one-to-one function
discrete function
continuous function
vertical line test
independent variable
dependent variable
function notation

GET READY for the Lesson

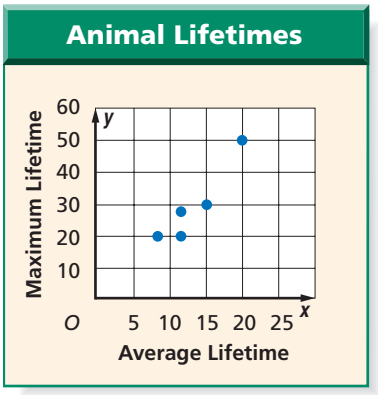
The table shows average and maximum lifetimes for some animals. The data can also be represented as the **ordered pairs** (12, 28), (15, 30), (8, 20), (12, 20), and (20, 50). The first number in each ordered pair is the average lifetime, and the second number is the maximum lifetime.

Animal	Average Lifetime (years)	Maximum Lifetime (years)
Cat	12	28
Cow	15	30
Deer	8	20
Dog	12	20
Horse	20	50

Source: *The World Almanac*

(12, 28)
average lifetime
maximum lifetime

Graph Relations You can graph the ordered pairs above on a *coordinate system*. Remember that each point in the coordinate plane can be named by exactly one ordered pair and every ordered pair names exactly one point in the coordinate plane.

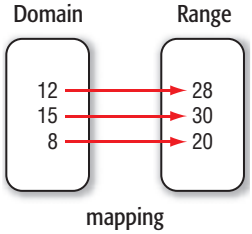


The graph of the animal lifetime data lies in the part of the Cartesian coordinate plane with all positive coordinates. The **Cartesian coordinate plane** is composed of the *x-axis* (horizontal) and the *y-axis* (vertical), which meet at the *origin* (0, 0) and divide the plane into four **quadrants**. In general, any ordered pair in the coordinate plane can be written in the form (x, y).

A **relation** is a set of ordered pairs, such as the one for the longevity of animals. The **domain** of a relation is the set of all first coordinates (x-coordinates) from the ordered pairs, and the **range** is the set of all second coordinates (y-coordinates) from the ordered pairs. The domain of the function above is {8, 12, 15, 20}, and the range is {20, 30, 28, 50}.

A **function** is a special type of relation in which each element of the domain is paired with *exactly one* element of the range. A **mapping** shows how the members are paired. A function like the one represented by the mapping in which each element of the range is paired with exactly one element of the domain is called a **one-to-one function.**

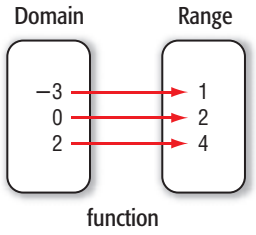
{(12, 28), (15, 30), (8, 20)}



William J. Weber

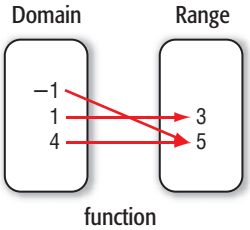
The first two relations shown below are functions. The third relation is not a function because the −3 in the domain is paired with both 0 and 6 in the range.

{(−3, 1), (0, 2), (2, 4)}



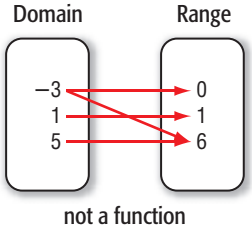
function

{(−1, 5), (1, 3), (4, 5)}



function

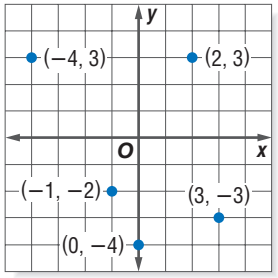
{(5, 6), (−3, 0), (1, 1), (−3, 6)}



not a function

EXAMPLE Domain and Range

1 **State the domain and range of the relation shown in the graph. Is the relation a function?**



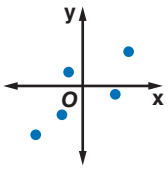
The relation is {(−4, 3), (−1, −2), (0, −4), (2, 3), (3, −3)}.
The domain is {−4, −1, 0, 2, 3}.
The range is {−4, −3, −2, 3}.

Each member of the domain is paired with exactly one member of the range, so this relation is a function.

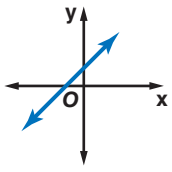
CHECK Your Progress

1. State the domain and range of the relation {(−2, 2), (1, 4), (3, 0), (−2, −4), (0, 3)}. Is the relation a function?

A relation in which the domain is a set of individual points, like the relation in Example 1, is said to be **discrete.** Notice that its graph consists of points that are not connected. When the domain of a relation has an infinite number of elements and the relation can be graphed with a line or smooth curve, the relation is **continuous.** With both discrete and continuous graphs, you can use the **vertical line test** to determine whether the relation is a function.

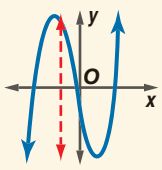
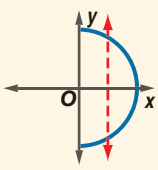


Discrete Relation



Continuous Relation

KEY CONCEPT *Vertical Line Test*

Words	If no vertical line intersects a graph in more than one point, the graph represents a function.	If some vertical line intersects a graph in two or more points, the graph does not represent a function.
Models		

Study Tip

Continuous Relations

You can draw the graph of a continuous relation without lifting your pencil from the paper.

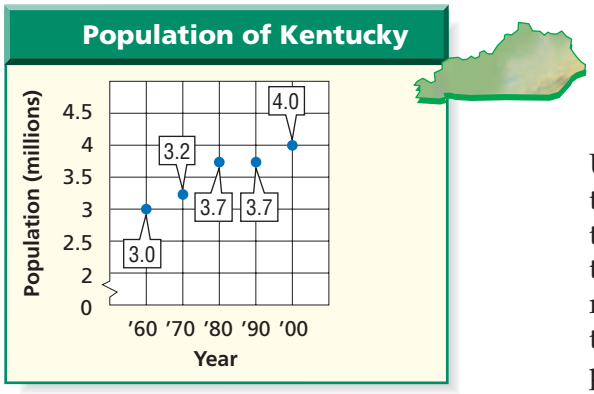
In Example 1, there is no vertical line that contains more than one of the points. Therefore, the relation is a function.

EXAMPLE Vertical Line Test

2 **GEOGRAPHY** **The table shows the population of the state of Kentucky over the last several decades. Graph this information and determine whether it represents a function. Is the relation *discrete* or *continuous*?**

Year	Population (millions)
1960	3.0
1970	3.2
1980	3.7
1990	3.7
2000	4.0

Source: U.S. Census Bureau



Use the vertical line test. Notice that no vertical line can be drawn that contains more than one of the data points. Therefore, this relation is a function. Because the graph consists of distinct points, the relation is discrete.

Study Tip

Vertical Line Test

You can use a pencil to represent a vertical line. Slowly move the pencil to the right across the graph to see if it intersects the graph at more than one point.

CHECK Your Progress

2. The number of employees a company had in each year from 1999 to 2004 were 25, 28, 34, 31, 27, and 29. Graph this information and determine whether it represents a function. Is the relation *discrete* or *continuous*?

Equations of Functions and Relations Relations and functions can also be represented by equations. The solutions of an equation in x and y are the set of ordered pairs (x, y) that make the equation true.

Consider the equation $y = 2x - 6$. Since x can be any real number, the domain has an infinite number of elements. To determine whether an equation represents a function, it is often simplest to look at the graph of the relation.

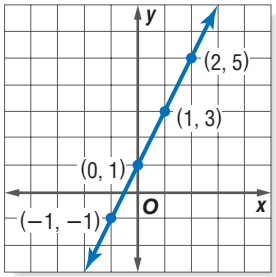
EXAMPLE Graph a Relation

3 **Graph each equation and find the domain and range. Then determine whether the equation is a function and state whether it is *discrete* or *continuous*.**

a. $y = 2x + 1$

Make a table of values to find ordered pairs that satisfy the equation. Choose values for x and find the corresponding values for y. Then graph the ordered pairs.

x	y
-1	-1
0	1
1	3
2	5



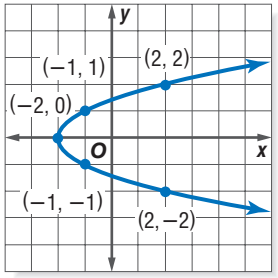
Since x can be any real number, there is an infinite number of ordered pairs that can be graphed. All of them lie on the line shown. Notice that every real number is the x-coordinate of some point on the line. Also, every real number is the y-coordinate of some point on the line. So the domain and range are both all real numbers, and the relation is continuous.

This graph passes the vertical line test. For each x-value, there is exactly one y-value, so the equation $y = 2x + 1$ represents a function.

b. $x = y^2 - 2$

Make a table. In this case, it is easier to choose y values and then find the corresponding values for x. Then sketch the graph, connecting the points with a smooth curve.

x	y
2	−2
−1	−1
−2	0
−1	1
2	2



Every real number is the y-coordinate of some point on the graph, so the range is all real numbers. But, only real numbers greater than or equal to −2 are x-coordinates of points on the graph. So the domain is $\{x|x \geq -2\}$. The relation is continuous.

You can see from the table and the vertical line test that there are two y values for each x value except $x = -2$. Therefore, the equation $x = y^2 - 2$ does not represent a function.

CHECK Your Progress

3A. Graph the relation represented by $y = x^2 + 1$.

3B. Find the domain and range. Determine if the relation is *discrete* or *continuous*.

3C. Determine whether the relation is a function.

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Reading Math

Functions Suppose you have a job that pays by the hour. Since your pay *depends* on the number of hours you work, you might say that your pay is a *function* of the number of hours you work.

When an equation represents a function, the variable, usually x, whose values make up the domain is called the **independent variable**. The other variable, usually y, is called the **dependent variable** because its values depend on x.

Equations that represent functions are often written in **function notation**. The equation $y = 2x + 1$ can be written as $f(x) = 2x + 1$. The symbol $f(x)$ replaces the y and is read "f of x." The f is just the name of the function. It is not a variable that is multiplied by x. Suppose you want to find the value in the range that corresponds to the element 4 in the domain of the function. This is written as $f(4)$ and is read "f of 4." The value $f(4)$ is found by substituting 4 for each x in the equation. Therefore, $f(4) = 2(4) + 1$ or 9. *Letters other than f can be used to represent a function. For example, $g(x) = 2x + 1$.*

EXAMPLE **Evaluate a Function**

4 **Given $f(x) = x^2 + 2$, find each value.**

a. $f(-3)$

$f(x) = x^2 + 2$	Original function
$f(-3) = (-3)^2 + 2$	Substitute.
$= 9 + 2$ or 11	Simplify.

b. $f(3z)$

$f(x) = x^2 + 2$	Original function
$f(3z) = (3z)^2 + 2$	Substitute.
$= 9z^2 + 2$	$(ab)^2 = a^2b^2$

CHECK Your Progress

Given $g(x) = 0.5x^2 - 5x + 3.5$, find each value.

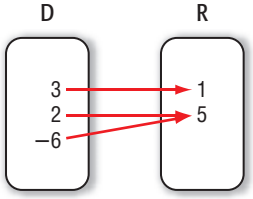
4A. $g(2.8)$ **4B.** $g(4a)$

CHECK Your Understanding

Examples 1, 2 (pp. 59–60)

State the domain and range of each relation. Then determine whether each relation is a function. Write *yes* or *no*.

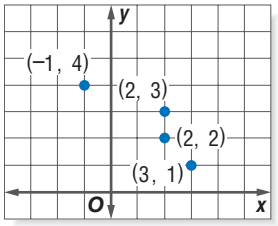
1.



2.

x	y
5	2
10	−2
15	−2
20	−2

3.



WEATHER **For Exercises 4–6, use the table that shows the record high temperatures (°F) for January and July for four states.**

4. Identify the domain and range. Assume that the January temperatures are the domain.

5. Write a relation of ordered pairs for the data.

6. Graph the relation. Is this relation a function?

State	Jan.	July
California	97	134
Illinois	78	117
North Carolina	86	109
Texas	98	119

Source: U.S. National Oceanic and Atmospheric Administration

Examples 2, 3 (pp. 60–61)

Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function and state whether it is *discrete* or *continuous*.

7. $\{(7, 8), (7, 5), (7, 2), (7, -1)\}$

8. $\{(6, 2.5), (3, 2.5), (4, 2.5)\}$

9. $y = -2x + 1$

10. $x = y^2$

Example 4 (p. 61)

11. Find $f(5)$ if $f(x) = x^2 - 3x$.

12. Find $h(-2)$ if $h(x) = x^3 + 1$.

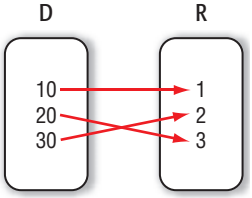
Exercises

HOMEWORK HELP

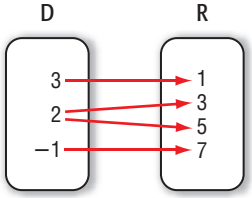
For Exercises	See Examples
13–28	1, 2
29–34	3
35–42	4

State the domain and range of each relation. Then determine whether each relation is a function. Write *yes* or *no*.

13.



14.



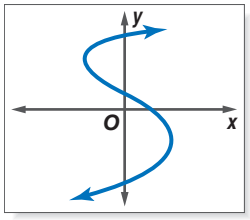
15.

x	y
0.5	−3
2	0.8
0.5	8

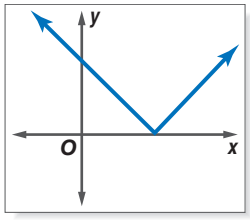
16.

x	y
2000	$4000
2001	$4300
2002	$4600
2003	$4500

17.

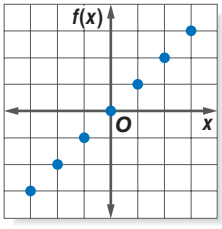


18.

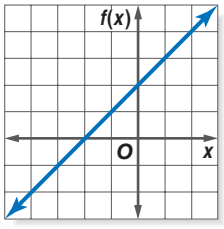


Determine whether each function is *discrete* or *continuous*.

19.



20.



21. $\{(-3, 0), (-1, 1), (1, 3)\}$

22. $y = -x + 4$

Graph each relation or equation and find the domain and range. Then determine whether the relation or equation is a function and state whether it is *discrete* or *continuous*.

23. $\{(2, 1), (-3, 0), (1, 5)\}$

24. $\{(4, 5), (6, 5), (3, 5)\}$

25. $\{(-2, 5), (3, 7), (-2, 8)\}$

26. $\{(3, 4), (4, 3), (6, 5), (5, 6)\}$

27. $\{(0, -1.1), (2, -3), (1.4, 2), (-3.6, 8)\}$

28. $\{(-2.5, 1), (-1, -1), (0, 1), (-1, 1)\}$

29. $y = -5x$

30. $y = 3x$

31. $y = 3x - 4$

32. $y = 7x - 6$

33. $y = x^2$

34. $x = 2y^2 - 3$

Find each value if $f(x) = 3x - 5$ and $g(x) = x^2 - x$.

35. $f(-3)$

36. $g(3)$

37. $g\left(\frac{1}{3}\right)$

38. $f\left(\frac{2}{3}\right)$

39. $f(a)$

40. $g(5n)$

41. Find the value of $f(x) = -3x + 2$ when $x = 2$.

42. What is $g(4)$ if $g(x) = x^2 - 5$?

Real-World Link

The major league record for runs batted in (RBIs) is 191 by Hack Wilson.

Source: www.baseball-almanac.com

SPORTS **For Exercises 43–45, use the table that shows the leading home run and runs batted in totals in the National League for 2000–2004.**

Year	2000	2001	2002	2003	2004
HR	50	73	49	47	48
RBI	147	160	128	141	131

Source: *The World Almanac*

43. Make a graph of the data with home runs on the horizontal axis and runs batted in on the vertical axis.

44. Identify the domain and range.

45. Does the graph represent a function? Explain your reasoning.

STOCKS **For Exercises 46–49, use the table that shows a company's stock price in recent years.**

46. Write a relation to represent the data.

47. Graph the relation.

48. Identify the domain and range.

49. Is the relation a function? Explain your reasoning.

Year	Price
2002	$39
2003	$43
2004	$48
2005	$55
2006	$61
2007	$52

GOVERNMENT **For Exercises 50–53, use the table below that shows the number of members of the U.S. House of Representatives with 30 or more consecutive years of service in Congress from 1991 to 2003.**

Year	1991	1993	1995	1997	1999	2001	2003
Representatives	11	12	9	6	3	7	9

Source: *Congressional Directory*

50. Write a relation to represent the data.

51. Graph the relation.

52. Identify the domain and range. Determine whether the relation is *discrete* or *continuous*.

53. Is the relation a function? Explain your reasoning.

54. AUDIO BOOK DOWNLOADS Chaz has a collection of 15 audio books. After he gets a part-time job, he decides to download 3 more audio books each month. The function $A(t) = 15 + 3t$ counts the number of audio books $A(t)$ he has after t months. How many audio books will he have after 8 months?

EXTRA PRACTICE
See pages 893, 927.
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H.O.T. Problems

55. OPEN ENDED Write a relation of four ordered pairs that is *not* a function. Explain why it is not a function.

56. FIND THE ERROR Teisha and Molly are finding $g(2a)$ for the function $g(x) = x^2 + x - 1$. Who is correct? Explain your reasoning.

Teisha
$g(2a) = 2(a^2 + a - 1)$
$= 2a^2 + 2a - 2$

Molly
$g(2a) = (2a)^2 + 2a - 1$
$= 4a^2 + 2a - 1$

57. CHALLENGE If $f(3a - 1) = 12a - 7$, find one possible expression for $f(x)$.

58. *Writing in Math* Use the information about animal lifetimes on page 58 to explain how relations and functions apply to biology. Include an explanation of how a relation can be used to represent data and a sentence that includes the words *average lifetime*, *maximum lifetime*, and *function*.

STANDARDIZED TEST PRACTICE

59. ACT/SAT If $g(x) = x^2$, which expression is equal to $g(x + 1)$?

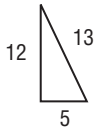
A 1

B $x^2 + 1$

C $x^2 + 2x + 1$

D $x^2 - x$

60. REVIEW Which set of dimensions represent a triangle similar to the triangle shown below?



F 7 units, 11 units, 12 units

G 10 units, 23 units, 24 units

H 20 units, 48 units, 52 units

J 1 unit, 2 units, 3 units

Spiral Review

Solve each inequality. (Lessons 1-5 and 1-6)

61. $|y + 1| < 7$

62. $|5 - m| < 1$

63. $x - 5 < 0.1$

64. SHOPPING Javier had $25.04 when he went to the mall. His friend Sally had $32.67. Javier wanted to buy a shirt for $27.89. How much money did Javier borrow from Sally? How much money did that leave Sally? (Lesson 1-3)

Simplify each expression. (Lessons 1-1 and 1-2)

65. $32(22 - 12) + 42$

66. $3(5a + 6b) + 8(2a - b)$

GET READY for the Next Lesson

PREREQUISITE SKILL Solve each equation. Check your solution. (Lesson 1-3)

67. $x + 3 = 2$

68. $-4 + 2y = 0$

69. $0 = \frac{1}{2}x - 3$

70. $\frac{1}{3}x - 4 = 1$